# Reassessment of the GZK cutoff in the spectrum of UHE cosmic rays in a universe with low photon-baryon ratio

Robin A.J. Booth

*Theoretical Physics, The Blackett Laboratory*
*Imperial College, Prince Consort Road, London SW7 2BZ, UK*

## ABSTRACT

A prediction of standard Big Bang cosmology is that the observed UHECR (ultra-high-energy cosmic rays) spectrum will exhibit a cutoff at the GKZ limit, resulting from interaction with the photons that constitute the cosmic microwave background. We show that for the Quasi-Static Universe (QSU) model, in which photon energy is an invariant in the cosmological reference frame, the photon number density in the universe today is a factor of $10^9$ less than in the standard model. As a consequence, the mean free path of UHECRs will exceed the horizon distance of the universe, rendering it essentially transparent to UHECRs. The QSU model therefore predicts that no cutoff will be observed in the UHECR spectrum.

*Subject headings:* cosmic microwave background — cosmic rays — cosmology: theory — scattering

## 1. Introduction

Experiments to measure the flux of UHECR have detected a number of events with energies $> 10^{20} eV$ (Watson 2000). These observations present something of a puzzle. There are no obvious mechanisms within our galaxy that could accelerate UHECRs to these energies. It is reasonable to postulate potential extra-galactic sources of UHECRs, e.g. quasars, but prevailing theory suggests that these particles should never reach the Earth because of interactions with the photons that constitute the cosmic microwave background radiation (CMBR). This prediction was made independently by Greisen (1966) and by Zatsepin & Kuzmin (1966), based on an analysis of the process $p + \gamma \rightarrow p + \pi$ (assuming that the majority of UHECRs at these energy levels are protons). This showed that UHECRs with energies $> 5 \times 10^{19}$ would exceed the energy threshold for photopion production (the GZK cutoff). Taking into account the collision cross section for the photopion reaction, and the CMBR

photon density, the mean free path for these UHECRs would be of the order of $10^{23}m$ (see Appendix A for analysis), implying that they could not have originated from sources outside our galaxy.

The present generation of UHECR experiments, such as AGASA and HiRes, are not able to generate a large enough sample from the relatively low flux of UHECRs at these high energies (De Marco, Blasi & Olinto 2003). The low statistical significance of the few observations of UHECRs with energies $> 10^{20}eV$ means that no firm conclusions can be drawn at this stage. Larger experiments are planned, such as the Pierre Auger Observatory (PAO) and EUSO, which will increase the number of detected events by 2 orders of magnitude. Only then will it be possible to to prove conclusively whether or not the predicted GZK cutoff feature is present in the UHECR spectrum. Nevertheless, it is reasonable to evaluate potential cosmological scenarios that might result in the detection of significant UHECR flux above the presumed GZK cutoff.

There have been suggestions that the observed UHECRs with energies $> 10^{20}eV$ might be due to Lorentz symmetry breaking (Kifune 1999; Magueijo & Smolin 2002), or deformation of Lorentz symmetry (Amelino-Camelia 2002). These predictions are based on a class of models that postulate deviations from standard special relativity for particles that have attained very high energies, such that $m/R < E/E_P$, where $m$ and $E$ are respectively the particle mass and total energy, and $E_P$ is the Planck energy. These are generally referred to as Double Special Relativity (DSR) models. The main criticism of this approach is that it requires either a preferred class of inertial observers, or an ad-hoc construction of the Lorentz deformation function to generate predicted effects that would result in a shift of the GZK cutoff.

An alternative solution to the GZK cutoff problem is proposed in the following sections, based on a significantly lower CMB photon number density that arises naturally from an alternative cosmological model (Booth 2002).

## 2. The Quasi-Static Universe model

The Quasi-Static Universe (QSU)is used here as a shorthand to refer to the paradigm described in Booth (2002). The main feature of this model is that the Planck scale is decoupled from the atomic scale conventionally used as the as the basis for our measurement system. The changes in the dynamical behaviour of the universe that result from this modification can not only account for most of the problems associated with the Big Bang model, but are also able provide a very simple explanation for the apparent acceleration of the

expansion rate of the universe that has been observed in various high redshift supernovae studies in recent years. One of the principal consequences of the QSU paradigm is that the atomic scale, as defined by the de Broglie wavelength of sub-atomic particles, is not an appropriate reference frame for measuring gravitational phenomena or the behaviour of photons. The QSU model is based on the postulate that the correct reference frame for these phenomena is in fact a cosmological frame based on the mass and size of universe as a whole. In such a frame, photons do not undergo any change in frequency, since as far as they are concerned, the universe is static. Hence, it is not meaningful to talk in terms of photons losing energy in this reference frame. In transforming from the cosmological reference frame to our conventional atomic frame, photons will be perceived to exhibit a redshift as the scale factor of the universe with respect to the atomic frame increases with time. The crucial difference between the QSU model and the conventional formalism is that the relationship $E_\gamma = h\nu$ no longer holds true for photons emitted at times $t < t_0$, where $t_0$ signifies the present time. The energy of such 'old' photons will remain constant, at the same value they possessed when they were first emitted. However their power, i.e. the energy transferred per unit time, will be reduced in proportion to their redshift, such that $W = W_0.\nu/\nu_0$ , where $W_0$ and $\nu_0$ are respectively the photon power and frequency at the time of emission. Clearly, such a modification to one the most fundamental equations in physics will have a very significant impact on any phenomena that involve redshifted photons, and in particular, the CMBR.

To understand the implications of the QSU model for the CMBR, we need to review the way in which the standard Planck black-body distribution law is applied. A summary of the standard derivation of this law is provided in Appendix B. Present day observations of the CMB give a value for the energy density of $U \simeq 4 \times 10^{-14} Jm^{-3}$, which from (B4) corresponds to a temperature of $T = 2.7°K$. Conventionally, the next step is to take this temperature and use equation (B6) to calculate the photon number density, giving a result of $N \simeq 4 \times 10^8 m^{-3}$. However, these formulae are only valid for black-body radiation *that is in equilibrium with its surroundings*. It would be perfectly correct to use these equations if we wished to deduce the photon number density for a black-body with this temperature *today*. In erroneously applying them in the context of the relic CBMR generated by the Big Bang, we are perpetuating the assumption inherent in going from (B3) to (B5) - that photon energy is always equal to $h\nu$.

In the QSU model, red-shifted photons do not lose energy, so it is not possible to determine the energy of an observed photon merely by measuring its frequency. It is also necessary to know the thermal history of the photon, i.e. its frequency when it was originally emitted. It is this that determines the energy of the photon. In order to obtain the correct result for the photon number density today that corresponds to an observed energy density,

we need to correct for the fact that the photon energy $h\nu$ applied in going from (B3) to (B5) should reflect the energy at the time of emission, or more precisely, its energy at the time it was in black-body equilibrium with its surroundings.

We therefore need to apply a factor of $T_{obs}/T_{equi}$ to the expression for photon number density in (B5), where of $T_{obs}$ is the observed absolute temperature of the CMBR today, and $T_{equi}$ is the temperature at the time of black-body equilibrium. Clearly we do not know this figure with any precision. However we can take an educated guess that is at least consistent with other observational data and with plausible models for primordial nucleosynthesis. One such model is the neutron decay variant of the Cold Big Bang (CBB), which predicts a maximum reaction temperature of $\approx 10^{10}K$, with an equilibrium temperature of the order of $\approx 10^9K$. Applying the correction factor to equation (B6) gives a calculated photon number density of $\approx 0.3m^{-3}$ - a factor of $\sim 10^9$ lower than for the standard Hot Big Bang (HBB) model. This is very close to the measured baryon number density of the universe, giving $\eta_\gamma \simeq 1$.

## 3. Implications for cosmological processes

Having established that the value of $\eta_\gamma$ in the QSU model is many orders of magnitude less than the conventionally accepted value, the natural question to ask is: how does this affect other cosmological processes?

### 3.1. Primordial nucleosynthesis

Arguably, one of the process most sensitive to changes in $\eta_\gamma$ is that of primordial nucleosynthesis. Applying the conventional value of $\eta_\gamma \simeq 2 \times 10^9$ to the standard model for HBB nucleosynthesis results in predicted element abundances that are in good accord with observational data. Relatively small changes in $\eta_\gamma$ will result in large changes to the predicted abundances, and would therefore appear not to be compatible with observations. However, it has been pointed out by Aguirre (2001) that, provided that appropriate changes are made to a range of initial parameters, including $\eta_\gamma$, it is possible to construct alternative models for primordial nucleosynthesis that will produce predicted element abundances that are in accord with observational data. One such model is the CBB, which takes a value of $\eta_\gamma \sim 1$ as one of its initial conditions. It is perhaps worth mentioning in passing that the neutron decay variant of the CBB model predicts a photon energy of $0.78MeV$, giving a baryon to photon energy ratio of $E_\gamma/E_B \simeq 1200$. Under the QSU scenario, this ratio should persist

from the nucleosynthesis epoch to the present day, and indeed, the observed value of $E_\gamma/E_B$ is very close to this value.

### 3.2. Hydrogen ionization

Another astrophysical measurement that is linked to $\eta_\gamma$ is the hydrogen ionization fraction. As the universe expands and cools, protons recombine with electrons to form neutral hydrogen when the energy of CMB photons falls below the hydrogen ionization energy threshold. The equilibrium ionization fraction $\chi_e$ as a function of temperature is give by the Saha equation

$$\frac{1-\chi_e}{\chi_e} = \frac{4\sqrt{2}\varsigma(3)}{\sqrt{\pi}\eta_\gamma}\left(\frac{T}{m_e}\right)^{\frac{3}{2}} e^{B/T} \tag{1}$$

from which it can be see that the ionization fraction is also dependent on $\eta_\gamma$. The ionization fraction can be expressed as a function of redshift using the relation $T = 2.73(1+z)K$. This is plotted in Figure 1 for a standard HBB cosmology with $\eta_\gamma = 2\times10^9$, and a CBB cosmology with $\eta_\gamma = 1$. This illustrates that a reduction in the photon to baryon ratio will cause recombination to take place at a much lower redshift, with the surface of last scattering for the CMB occurring at $z \simeq 2400$, as compared to $z \simeq 1100$ for the standard model.

At first sight it might be expected that such a change should inevitably result in very significant modifications to the observed CMB angular power spectrum. However, an analysis of the dynamics of the two cosmological models reveals that the linear expansion of a CBB universe, in comparison to the rapid initial expansion rate of a standard model HBB universe, results in the epoch of creation of the last scattering surface being similar, in terms of coordinate time, for both cosmologies. Further detailed numerical analysis is required, using a package such as CMBFAST, in order to accurately determine the nature of the impact on the predicted CMB angular power spectrum arising from a $\eta_\gamma = 1$ cosmological model. Nevertheless, initial indications are that such a model may well be consistent with the observed CMB angular power spectrum.

### 3.3. Reassessment of the GZK cutoff

The interaction between CMB photons and UHECRs is dependent on three factors: the photon energy, the capture cross-section for the photon-proton collision, and the photon

number density. We shall now examine how each of these factors varies in a CBB universe.

From (A4) it might be anticipated that if each CMB photon in a QSU is a factor of $\sim 10^9$ more energetic than its counterpart in a standard HBB universe, then the threshold energy for the photon-proton reaction will be reduced from $\sim 10^{20}eV$ to $\sim 10^{11}eV$. However, this is unlikely to be the case here since energy transfer from the CMBR photon to the proton will be dependent on the photon power, rather than its total energy. Since the photon power is, as in the standard case, proportional to $h\nu$, there will be no change in the GZK cutoff threshold under the QSU scenario.

The next factor to consider is the collision cross-section for the photon-proton interaction. As discussed in Appendix A, this varies by a factor of $\sim 5$ between energies corresponding to the main $\Delta^+$ resonance peak and higher photon energy levels. It is not, therefore, a significant factor in differentiating between the two cosmological scenarios.

Finally, we must consider the impact of the CMB photon number density on the mean free path of the UHECRs. Under the QSU model, the CMBR photon number density is a factor of $\sim 10^9$ less than for the HBB model. From (A6), taking a value for $\sigma = 100\mu$barn, and $n = 0.2m^{-3}$, gives a value for the mean free path of $\lambda = 10^{32}m$. This is several orders of magnitude greater than the horizon distance of the universe. The universe is therefore effectively transparent to UHECRs, and the QSU model predicts that there will be no observable reduction in the flux of UHECRs as a result of the GZK cutoff.

## 4. Conclusions

The reduced CMB photon number density predicted by the QSU model provides a natural explanation for the observed UHECR flux with energies $> 10^{20}eV$. However, the statistical significance of the measurements obtained from the existing AGASA and HiRes experiments is still to low to provide conclusive evidence for the presence or absence of the GZK cutoff feature. For this, we will have to wait for the results from the next generation of UHECR experiments, such as PAO and EUSO.

## A. The photon-proton reaction

The principal reaction contributing to the attenuation of UHECRs is the interaction with CMB photons

$$p+\gamma \;\rightarrow\; \Delta^{+} \rightarrow p+\pi^{0} \tag{A1}$$
$$p+\gamma \;\rightarrow\; \Delta^{+} \rightarrow n+\pi^{+} \tag{A2}$$

The reaction will proceed when the combined centre-of-mass energy of the proton and photon is equal to or greater than the sum of the pion and proton (or neutron) mass, which can be expressed as

$$m_p m_\pi + \frac{m_\pi^2}{2} \leq q\left(\sqrt{p^2+m_p^2} - p\cos\theta\right) \tag{A3}$$

where $q$ is the photon momentum along the $x$-axis and $p$ is the momentum of the proton hitting the photon at an angle of $\theta$ in the $xy$ plane. Since the pion mass is much smaller than the proton (or neutron) mass, this expression can be simplified to

$$E_p - p\cos\theta \geq \frac{m_p m_\pi}{q} \tag{A4}$$

For a thermal gas of relativistic bosons $\langle q\rangle \sim 2.7T$, and with $T_{CMBR} \simeq 2.7K$, corresponding to an energy of $2.3\times10^{-4}eV$, inserting the pion and proton rest masses gives a cut-off energy of

$$E_p \sim 10^{20}eV \tag{A5}$$

This defines the Greisen Zatsepin Kuzmin (GZK) limit.

The mean free path for this reaction is given by

$$\lambda = \frac{1}{n\sigma} \tag{A6}$$

where $\sigma$ is the capture cross-section for the $p\gamma_{CMBR}$ reaction, and $n$ is the number density of CMBR photons. $\sigma$ varies from a maximum of $\sim 500\mu$barn at the main $\Delta^+$ resonance to a level of $\sim 100\mu$barn at higher energy levels (see for example (Mücke et al 1999)).

## B. Black Body Radiation

Planck's black-body distribution law can be derived in two steps. First, by considering the number of radiation modes that can be supported in a black-body cavity of unit volume, an expression for the mode density can be obtained

$$dN(\nu) = \frac{8\pi\nu^2}{c^3}d\nu \tag{B1}$$

The energy density as a function of frequency can then be calculated by multiplying the mode density by the average energy per mode. In classical terms this would simply be $kT$, where $k$ is the Boltzmann factor and $T$ is the absolute temperature. However, the quantum nature of photons results in the probability distribution being skewed, so that the correct mean energy per mode must be calculated from the Boltzmann distribution, giving

$$\bar{E} = \frac{h\nu}{e^{h\nu/kT} - 1} \tag{B2}$$

Combining (B1 and (B2) we obtain the final form of Planck's law for the energy density of black-body radiation as a function of frequency

$$dU(\nu) = \frac{8\pi h\nu^3}{c^3}.\frac{d\nu}{e^{h\nu/kT} - 1} \tag{B3}$$

The total energy density per unit volume is then simply obtained by integrating (B3) to give

$$U = \frac{8\pi^5 k^4 T^4}{15c^3h^3} \tag{B4}$$

The final expression for the photon number density as a function of frequency is merely (B3) divided by the photon energy, $h\nu$

$$dN(\nu) = \frac{8\pi\nu^2}{c^3}.\frac{d\nu}{e^{h\nu/kT} - 1} \tag{B5}$$

The photon number density per unit volume is therefore given by

$$N = \int_0^\infty \frac{8\pi\nu^2}{c^3}.\frac{d\nu}{e^{h\nu/kT} - 1} \tag{B6}$$

$$= \frac{16\pi k^3 T^3 \zeta(3)}{c^3h^3} \tag{B7}$$

## REFERENCES


Aguirre, A. 2001, Phys.Rev. D64, 083508

Amelino-Camelia, G. 2002, Preprint: astro-ph/0209232

Booth, R.A.J. 2002, Preprint: gr-qc/0203065

De Marco, D., Blasi, P., Olinto, A.V. 2003, Preprint: astro-ph/0301497

Greisen, K. 1966, Phys. Rev. Let. 16 , 748

Kifune, T. 1999, ApJ, 518, L21

Magueijo, J. and Smolin, L. 2002, Preprint: gr-qc/0207085

Mücke, A., Rachen, J.P., Engel, R., Protheroe, R.J., Stanev, T. 1999, PASA 16(2)

Watson, A.A. 2000, Phys. Rep., 333, 329

Zatsepin, G.T. and Kuzmin, V. A. 1966 Sov. Phys. JETP Lett. 4, 78

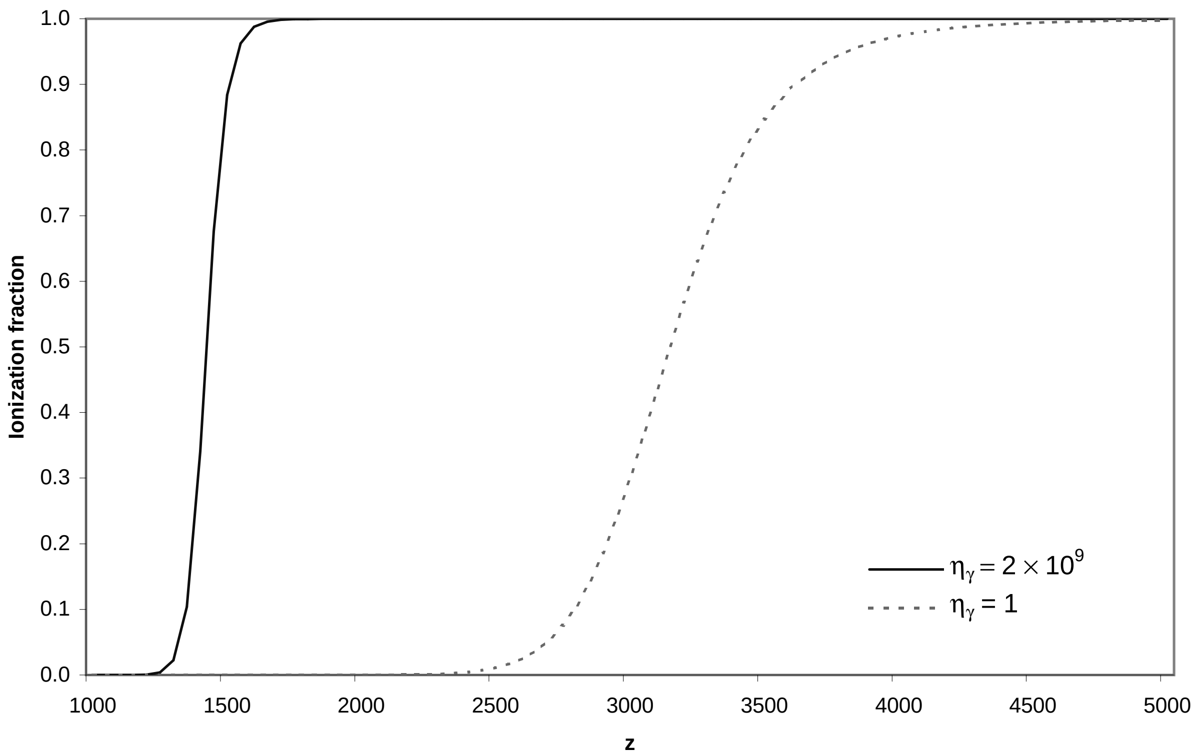


Fig. 1.— Ionization fraction as a function of red-shift